When More is Less: Limited Consideration*

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Abstract

There is well-established evidence that decision makers consistently fail to consider all available options. Instead, they restrict their attention to only a subset of alternatives and then undertake a more detailed analysis of this reduced set. This systematic lack of consideration of available options can lead to a "more is less" effect, where excess of options can be welfare-reducing for a decision-maker (DM). Building on this idea, we model individuals who might pay attention to only a subset of the choice problem presented to them. Within this smaller set, a DM is rational in the standard sense, and she chooses the maximal element with respect to her preference. We provide a choice theoretical foundation for our model. In addition, we show to which alternatives are revealed preferred to which, and discuss welfare implications.

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1 Introduction

According to Food Marketing Institute, in the USA, an average supermarket carries more than 40,000 products, this is a reflection of the ongoing tendency of increasing the number of available options offered to consumers.[1] According to the classical economic theory models, this abundance of variety is beneficial for consumers; but in reality, too many options tend to overwhelm consumers, and thus lead them to neglect some products that are available.[2] This phenomenon is known as "choice overload" in psychology literature. In this paper, we show how to make welfare analysis under the possibility of choice overload, as well as provide choice theoretical foundation for this phenomenon.

When consumers are overwhelmed by the abundance of options, every product competes for consumers' attention; and as the number of the alternatives increases, the competition gets more severe. The marketing literature calls the set of alternatives that prevails on the competition for the consumers' attention the consideration set (Wright and Barbour 1977). To deal with choice overloads, consumers uses many heuristics to generate consideration sets; we list some of these heuristics to illustrate that the formation of consideration sets varies significantly.

- **Top N**: The decision-maker (DM) pays attention to the top N elements according to some ranking such as the amount of advertisement or the order of internet searches (see Rubinstein and Salant (2006) and Rubinstein and Salant (2012)). For example, a DM considers all the items appearing in the first page of search results and overlooks the rest.

- **Shortlisting**: (Manzini and Mariotti 2007) From every choice problem, the DM creates a shortlist of alternatives that are undominated according to a an asymmetric (possibly incomplete and/or cyclic) binary relation. Any alternative outside of the shortlist will be disregarded.

- **Top on Each**: There are several rankings, and the DM considers only the top N elements in each ranking. For instance, one may consider *only* the cheapest car, the most fuel efficient car, and the most advertised car in the market.[3]

[1] For instance, nowadays a shopper in a supermarket needs to select from 285 varieties of cookies, 85 flavors and brands of juices, 230 different soups, and 275 varieties of boxed cereal (Schwartz 2005).

[2] In financial economics, (see e.g. Huberman and Regev (2001)), it is known that investors make investing decision based on a limited number of all the available options, possibly good ones. Similar examples can be found in job search (Richards et al. 1975), university choice (Laroche et al. 1984, Rosen et al. 1998), and airport choice (Basar and Bhat 2004).

[3] This behavior is often called "all or nothing" or "extreme seeking" behavior (Gourville and Soman 2005).

Another example is that we only consider the top N job candidates in each field to hire an assistant professor.

- **Categorization**: (Manzini and Mariotti 2012) The DM categorizes all alternatives on the basis of some criterion, for example "similarity".[4] These categories can be (partially) compared. These comparisons are summarized by an asymmetric (possibly incomplete) binary relation. The DM considers only those belonging to an undominated category of options.

- **Rationalization**: (Cherepanov et al. 2013) Rationalization is the necessity to provide a logical explanation, avoiding the true reasons for the behavior. The DM only considers alternatives she can rationalize to choose. To do so, she finds one of the subjectively appealing rationales[5] to herself (and/or her family, society) that ranks that alternative as the best course of action given the set of alternatives. For example, a consumer considers a Subaru which is the best car among symmetrical all-wheel drive cars even though she does not care about this feature.

- **Narrowing Down**: The DM narrows down the size of the considered alternatives by considering only options that meet certain criteria. For example, the DM considers all products appearing in the search result if the total number is n or less. Otherwise, the DM adds another keyword to narrow down her search.

In this paper, we assume that once the consideration set is formed, consumers are able to maximize their well-defined preference within their consideration set. Reutskaja et al. (2011), a recent experimental paper, provides evidence for this assumption by utilizing both eye tracking and choice data. They find that subjects are quite adept at optimizing within the set of items that they see (they call it as "seen set").

Our aim is to uncover preferences from solely from observed choices. Choice overload is the outcome of the decision maker's cognitive limitations, it thus cannot be directly observed. Given the examples above, there are many ways of constructing consideration sets under choice overload. One can commit to a particular consideration set formation and study the revealed preference implications of such model. However, this approach is not going to be fruitful when we do not directly observe the way that consideration sets are constructed. Instead, here we impose a property on consideration set to capture the idea of competition among products. This is inline with the idea that increasing the number of alternatives can sometimes result

[4] See Smith et al. (1998) and Pothos (2005).

[5] As opposed to Top N, a rationale could be incomplete.

in choice overload. All the examples above satisfy this property. Hence a revealed preference result based on this property will be applicable to all the examples above.

Our property, *Competition Filter*, formalizes the idea that if a product grabs the consumer's consideration in a large supermarket (high choice overload), then it will grab her attention in a small convenience store with fewer rivals (low choice overload).[6] This property intuitively captures the idea of competition, and that competition is fiercer with more rivals. For instance, the eye-tracking study of Reutskaja et al. (2011) provides supporting evidence for our property by showing that as the number of alternatives increase, the frequencies of the least looked parts, such as the right bottom corner, decrease.

Our approach is similar to the one on Masatlioglu et al. (2012). Similar to ours, they also impose a property on consideration sets rather than focusing a particular formation of consideration sets. Their property, *Attention Filter*, is based on the idea of unawareness: if a consumer is not only unaware of a particular product but she is also unaware that she overlooks that product, then, her consideration set stays same if that product is removed. While their property is appealing in terms of unawareness, it will be violated by examples such as Categorization, Rationalization and Narrowing Down. Hence, their revealed preference result is *not* applicable for these examples. In addition, their property allows for a DM that considers *everything* in a bigger set but not in a smaller subset of it. Hence, their property is orthogonal to the idea of choice overload. Given that two properties are distinct, our paper complements their paper.[7]

We call our choice model as *Overwhelming Choice (OC)*. In our model, the DM has a well-defined preference and is maximizing her preference within her consideration set; where the formation of consideration sets satisfies the Competition Filter property. It is natural to ask for the falsifiability of our model. Our characterization result provides necessary and sufficient conditions on the observed choices such that even if the consideration set is not observable or the heuristic is not known, it is still possible to conclude as an analyst that observed choices are consistent with the overwhelming choice model. Surprisingly, it turns out that our model is characterized through one testable property of choice, which is a version of the weak axiom of revealed preference (WARP) in the classical choice setting.

[6]This *consistency* property is the same as Sen's α property. But the important difference is that Sen's property is on the choices while ours is on the consideration set.

[7]Given these two properties, it is natural to consider a model where consideration sets being both the competition filter and of the attention filter at the same time. There is an interesting interaction between these properties. Indeed, putting together our axiom and the axiom of Masatlioglu et al. (2012) does not deliver a characterization for such a model. Because of space limitations, we do not provide a characterization but the result will be available upon request.

As opposed the WARP, our condition distinguishes between "being feasible" and "being considered," which is a key difference when making behavioral inference on observed choices. Our axiom relaxes the WARP by replacing being-feasible with being-considered in the WARP. According to the WARP (without indifference), if x is chosen while y is available, x is revealed preferred to y so y should not be chosen in any other circumstance where x is available. This is not necessarily true under limited consideration as y may be chosen over x if x is not considered. Instead, our axiom understands x is preferred to y only when x is chosen while it is certain that y is considered. Then, the axiom requires that y should never be chosen when we are sure that x is considered.

Although our characterization theorem shows how to test the OC model, it does not directly deliver information about preferences unlike the classical choice theory (a la Samuelson).[8] In the classical theory, the chosen alternative is (weakly) revealed to be preferred to any other available alternative. In OC, being chosen is not enough to make that inference since in our model there is a distinction between "being feasible" and "being considered," as previously mentioned. Nevertheless, we are able to show how to make welfare inferences (and also derive information about preferences) when limited consideration is present. Having information about which elements are being considered is vital to make inferences about preferences; it turns out that "choice reversals" is the criterion required to make inferences about preference, as it reveals consideration of certain elements, which is necessary to derive any conclusion about preferences. Choice reversals refer to the situations where choices from a small set and a larger set are "inconsistent" in the classical sense (i.e. some element is chosen from a set, and then not chosen from a subset of it still containing the choice from the superset).

To illustrate this point, consider two nested menus, $T \subset S$, both including x and y. Assume that x is the chosen alternative from T, but from the larger set, S, the chosen one is y. Firstly, x should be considered in T, and y should be considered in S since they are chosen in those sets. Competition property implies that since y is considered in S, then it must be considered in its subsets, in particular in T, i.e. both x and y should be considered in T. Hence, the choice of x in T reveals that x is better than y (revealed preference). Since in the larger set, S, the better alternative x is not chosen, we can infer that x is not considered in S. Hence, having more option leads to not considering the better option, and choosing the suboptimal one. Although this example illustrates, the necessity of the choice reversal, we prove that choice reversal is both necessary and sufficient condition for revealed preference. Hence, observing choice reversal will allow us to conclude the negative consequences of having more

[8] See Samuelson (1938).

alternatives.

The remainder of the paper will be structured as follows Section 2 formally defines and discusses the relevant competition filters. We also provide a characterization the choice with limited consideration model for functions and linear orders and discuss the revealed preference implications on this framework. Section 3 discusses the related literature, and finally section 4 concludes.

2 Choice with Limited Consideration

Let X be an arbitrary non-empty set and $\mathcal{X}$ be the set of all non-empty subsets of X. $\Gamma(S)$ denotes the consideration set under $S \in \mathcal{X}$; that is, the set of alternatives is considered when the DM is facing feasible set S. We assume that $\Gamma(S)$ is always a subset of S, as the DM can only consider options that are available. Note that we do not assume any prior knowledge about how consumers form their consideration set since there are many ways to construct consideration sets, as we discuss with the examples in the Introduction. That is why we do not commit to a specific formation of consideration sets. Instead, we directly impose a condition on the structure of consideration sets that not only captures the idea of more is less, but it is also is satisfied by many different intuitive consideration decisions (see examples in the Introduction).

Definition. *A function $\Gamma : \mathcal{X} \to \mathcal{X}$ is called a **competition filter** if for all $x \in S \subset T$ and $x \in \Gamma(T)$ then $x \in \Gamma(S)$.*

Competition filters capture systematic failures to consider all available options based on the "more is less" phenomenon. In a nutshell, when the size of the opportunity set gets larger, choice overload becomes more pronounced and consumers tend to overlook more options. This idea is consistent with the Miller (1956) findings of the limited amount of information that DMs can process, and with the empirical evidence which shows the complexity of a decision process as a function of size of the menu. In addition, analyzing consideration as a function of the size of the menu is common on the marketing literature in the works of Shugan (1983) or Hauser and Wernerfelt (1990), where as both the number of options and the information about options increase, DMs consider fewer alternatives and process a smaller fraction of the overall information available regarding each alternative.

We now describe the behavior of our DM: she picks her most preferred alternative within her consideration set, not the entire feasible set. Formally, let c be a choice function: $c : \mathcal{X} \to X$ and $c(S) \in S$ for all $S \in \mathcal{X}$. Let $\succ$ be a complete, transitive and antisymmetric binary relation (a linear order) over X and denote the best element in S with respect to $\succ$ by $\max(\succ, S)$. Given this, we define our model: *Overwhelming Choice*.[9]

Definition. *A choice function c is an Overwhelming Choice (OC) if there exists a strict linear order $\succ$ and a competition filter Γ such that*

$$c(S) = \max(\succ, \Gamma(S))$$

Occasionally, we will say that $(\Gamma, \succ)$ represents c. We also mention that $\succ$ represents c, which means that there exists some competition filter Γ such that $(\Gamma, \succ)$ represents c.

Our characterization result is concerned with finding necessary and sufficient conditions for the type of choice behavior is consistent with our model; in other words we answer the question on how one could test whether choice data is consistent with the overwhelming choice model. It turns out that the model can be simply characterized by one observable property of choice, just like WARP in the classical choice model.

Before we state our behavioral postulate, recall the standard Weak Axiom of Revealed Preference (WARP). WARP requires that every set S has the "best" alternative b^* (for choice functions), that is, b^* must be chosen from a budget set T whenever b^* is available and the choice from T lies in S. Formally,

> **WARP.** For any nonempty S, there exists $b^* \in S$ such that for any T including b^*,
> $$\text{if } c(T) \in S \text{ then } c(T) = b^*.$$

Unlike the standard theory, we must distinguish between "being feasible" and "being considered" due to choice overload. Therefore, we cannot conclude that b^* is chosen from T without confirming that b^* is considered. So, when can we infer that b^* is considered? Since Γ is a competition filter, in order for b^* to be considered at T, b^* must chosen from a superset of T. That is, if $b^* = c(T')$ for some $T' \supset T$, then $b^* \in \Gamma(T')$ since a necessary condition for choice is that the b^* is considered. Since Γ is a competition filter, $b^* \in \Gamma(T)$. This discussion suggests WARP under choice overload (WARP-CO), which is a weakening of WARP:

[9] We consider strict preferences in the body of the paper and show on the Appendix that the analysis can be extended to the weak order case where we allow for indifference.

(A1) WARP-CO. For any nonempty S, there exists $b^* \in S$ such that for any T including b^*,

$$\text{if} \quad \begin{array}{l} (i)\ c(T) \in S,\ \textit{and} \\ (ii)\ b^* = c(T') \text{ for some } T' \supset T \end{array} \qquad \text{then } c(T) = b^*$$

While WARP-CO is much weaker than WARP, only three observations can falsify it. For example, consider the following choice pattern:

$$c(\{x, y, z, t\}) = y, \quad c(\{x, y, z\}) = x \text{ and } c(\{x, y\}) = y.$$

To see how WARP-CO rules out the example above, take $S = \{x, y\}$; we will see that no element satisfies the condition of b^* for S. Either x or y should obey the condition (i.e. have the role of b^*) in the axiom for S. Suppose that $b^* = y$, and consider $T' = \{x, y, z, t\}$, and $T = \{x, y, z\}$. Since $y = c(\{x, y, z, t\})$ and $c(\{x, y, z\}) \in S$, if c satisfies WARP-CO then $y = c(\{x, y, z\})$, which is not the case. If $b^* = x$, then consider $T' = \{x, y, z\}$ and $T = \{x, y\}$. Since $c(\{x, y, z\}) = x$, then WARP-CO would require $x = c(\{x, y\})$, which is not true either. So there is no element in S that satisfies the condition for b^*. Therefore, the above example violates WARP-CO for $S = \{x, y\}$. Note that the axiom applies to any set of alternatives so it rules out more than this example.

To put this example in the context of our model, $c(\{x, y, z, t\}) = y$ reveals y is considered under $\{x, y, z, t\}$; this in turn implies that the DM must consider y in $\{x, y, z\}$ from the competition filter property. Hence $c(\{x, y, z\}) = x$ requires that she prefers x to y ($x \succ y$) since she considers y and chooses x. By a similar argument, $c(\{x, y, z\}) = x$ and $c(\{x, y\}) = y$ imply she prefers y to x ($y \succ x$) since she considers x and chooses y. This is a contradiction, since it would imply a cycle of size 2.[10]

The argument above hints that the choice reversals in our model directly reveals her preference: whenever her choices from a small set and a larger set are inconsistent, the former reflects her true preference under this framework. Formally, for any distinct x and y, define the following binary relation:

$$xPy \text{ if } x = c(S) \text{ and } y = c(T) \text{ such that } \{x, y\} \subseteq S \subset T \tag{1}$$

If we observe y being chosen from a larger menu, we infer that y must be considered in any subset of the menu including y (competition filter). Since x is chosen from a smaller menu containing y, then x must be preferred to y by the DM. This is a direct

[10] As we will see in Lemma 1, WARP-CO does not allow cycle of any size.

revelation. In addition, we can also conclude that she prefers x to z if xPy and yPz for some y, even when xPz does not hold (i.e. we do not observe a choice reversal from z to x). This is an indirect revelation. To denote both direct and indirect revelations, we use P_T, which is the transitive closure of P.

The above discussion makes it clear that the acyclicity of P_T is necessary condition for the revealed preference. Before we investigate this relationship, we first identify the link between WARP-CO and the acyclicity of P_T, which is the only behavioral postulate needed to characterize our *overwhelming choice* model.

Lemma 1. *A choice function c satisfies WARP-CO if and only if P_T is acyclical.*

Proof. First, we show that c WARP-CO implies that P (and thus P_T) is acyclical by contraposition. Assume that $x_n P x_{n-1} P \cdots P x_1 P x_n$ occurs. Then there exists no element in $\{x_1, \ldots, x_n\}$ serving the role of b^* in the axiom. For example, x_k cannot be b^* since $x_{k+1} P x_k$, i.e. there exist $S_k \subset T_k$ with $\{x_k, x_{k+1}\} \subset S_k \subset T_k$ such that $x_{k+1} = c(S_k)$ and $x_k = c(T_k)$. To show that P_T acyclical implies that c satisfies WARP-CO, take $S \in \mathcal{X}$, since P is acyclical, there exists at least one alternative in S which is undominated with respect to P. Then it is routine to check that any of them serves the role of b^* in the axiom. $\square$

The following theorem shows that our overwhelming choice model is captured by a single behavioral postulate: WARP-CO.

Theorem 1. *A choice function c satisfies WARP-CO if and only if c is an overwhelming choice.*

Proof. The if-part is a direct implication of Lemma 1. If c violates WARP-CO, its revealed preference has a cycle. Let us prove the only-if part. By Lemma 1 and the existence of a linear order that is an extension of a partial order on a nonempty X, there is a preference that includes P_T. Take such a preference arbitrarily and define

$$\Gamma^m(S) = \{x \in S | \ x = c(T) \text{ for some } T \supseteq S\}$$

By construction, if $x \notin \Gamma^m(S)$ then there exists no T such that $x = c(T)$ and $S \subseteq T$. This implies that there exists no T such that $x = c(T)$ and $S \cup \{y\} \subseteq T$. Hence, $x \notin \Gamma^m(S \cup y)$ for all y, therefore Γ^m is a competition filter.

For any S, we need to show that $c(S)$ is the $\succ$-best element in $\Gamma^m(S)$. Note that $c(S) \in \Gamma^m(S)$. Let $x \neq c(S)$ and $x \in \Gamma^m(S)$. Then there exists $T \supseteq S$ such that $x = c(T)$. By construction of $\succ$, it must be $c(S) P x$, so $c(S) \succ x$. Hence $(\Gamma, \succ)$ represents c. $\square$

This theorem states that it is possible to test our model non-parametrically by using a revealed-preference technique even when the consideration sets themselves are unobservable.

As we previously mentioned, while the theorem provides a test for our model, it does not deliver any information about the decision-maker's preferences or consideration sets. As opposed to the classical choice theory, there are multiple possible preference rankings which can rationalize the same choice pattern.[11] Here, we use the notion of the revealed preference introduced by Masatlioglu et al. (2012). They define the revealed preferences as follows. If x is ranked above y in *every* possible representation, x is **revealed preferred** to y.

If one wants to know whether x is revealed preferred to y, it seems to be necessary to check for every possible representation whether it represents her choice or not, which is not practical especially when there are many alternatives. We shall now provide a characterization of her revealed preference.

As we have seen before, it is sufficient to have xP_Ty to conclude that x is revealed preferred to y. A natural follow-up question is whether there is some revealed preference that is not captured by P_T. The next proposition states that the answer is no: P_T is the revealed preference.

Proposition 1. *Let c be an overwhelming choice. Then x is revealed to be preferred to y if and only if xP_Ty.*

Proof. We have already illustrated the if-part. To see the only-if part, take any pair of x and y without xP_Ty. Then there exists a preference $\succ$ including P_T and $y \succ x$ since P_T is transitive. By the proof of Theorem 1, $(\Gamma^m, \succ)$ represents c. Since $y \succ x$, by definition, x cannot be revealed to be preferred to y. $\square$

In the classical theory, "more is always better": a bigger budget set is always welfare enhancing. That is, $S \subset T$ implies $c(T) \succsim c(S)$. In our model, we find that "more is sometimes less": a smaller selection is sometimes welfare enhancing. We indeed identified the cases when this happens. When the choices from smaller and

[11] To illustrate this, consider the choice function with three elements exhibiting a cycle:

$$c(\{x,y,z\}) = y, \quad c(\{x,y\}) = x, \quad c(\{y,z\}) = y, \quad c(\{x,z\}) = z.$$

One possibility is that her preference is $x \succ y \succ z$ and she overlooks x both at $\{x,y,z\}$ and $\{x,z\}$. Another possibility is that her preference is $z \succ x \succ y$ and she does not pay attention to x only at $\{x,y,z\}$. Consequently, we cannot determine which of them is her true preference from her choice data. On the other hand, both of the preferences rank x above y. Therefore, if these two pairs are the only possibilities, we can unambiguously conclude that she prefers x to y.

larger selections do not match ($c(S) \neq c(T)$) and the choice from the larger menu is available in the smaller set ($c(T) \in S$), the smaller selection is always preferred to the larger selection ($c(S) \succ c(T)$). Hence, a smaller selection S is *strictly welfare enhancing* over T if $c(T) \in S \subset T$ and $c(S) \neq c(T)$. We can even provide a stronger condition for when more is less by utilizing our revealed preference result (Proposition 1).

Corollary 1. *More is less if* $c(S)P_T c(T)$ *and* $S \subset T$.

This corollary has two important implications. First, as opposed to the classical theory, even when choices satisfy WARP, we cannot say "more is always better." The reason is that her choices reveal nothing about her preferences, hence one cannot conclude whether smaller or larger selections are welfare enhancing. Second, in our model, the smaller sets is not *always* welfare enhancing. For instance, if $c(T)$ does not belong to S, then T could be revealed to be welfare enhancing over S. Especially, this happens when $c(T)$ is revealed to be preferred to any alternative in S.

3 Related Literature

Our model is related to a growing body of work on sequential elimination procedure. Manzini and Mariotti (2007) provide a model where a DM sequentially eliminates inferior alternatives according to asymmetric binary relations (rationales) until only one alternative remains as the final choice. The order of rationales are fixed. Manzini and Mariotti (2007) deliver a very simple characterization when there are only two rationales (shortlisting). Apesteguia and Ballester (2013) provide a characterization for any arbitrary number of *acyclical* rationales and link this procedure with other seemingly different procedures.[12] Dutta and Horan (2015) study how to identify rationales in shortlisting procedure from observed choice behavior.[13] The differences between our model and shortlisting (two rationales) are two folded: (i) our second stage preference is assumed to be complete and transitive, and (ii) our competition filter allows much richer structure compared to their set of surviving alternatives. If there are more than two rationales, the set of surviving alternatives might violate our property.[14]

[12] Mariotti and Manzini (2012) also provide a characterization for any arbitrary number of rationales.

[13] Au and Kawai (2011), Bajraj and Ülkü (2015), Houy (2007), Houy and Tadenuma (2009), Mariotti and Manzini (2012), Mandler et al. (2012), Garcia-Sanz and Alcantud (2015), Tyson (2013) also study variations of the sequential elimination procedure.

[14] There are competition filters that cannot be expressed as the set of surviving alternatives according to a set of rationales.

Our model is also closely related to *categorization* of Manzini and Mariotti (2012) and *rationalization* of Cherepanov et al. (2013). Both of them follow the two-stage choice process: in the first stage, a decision maker eliminates some of then alternatives, and in the second stage she maximizes her preference among the alternatives surviving after the first stage. Each model commits a particular consideration set formation: categorization and rationalization, respectively. As opposed these models, in our model, there is no single story why the decision maker does not consider all available alternatives in our model (see examples provided in Introduction). In addition, unlike our model, these models implicitly assume that a DM considers all feasible alternatives at the first stage and intentionally eliminates several alternatives. Therefore, their stories are not applicable to cases where the source of limited consideration is unawareness of some alternatives. It turns out that there is rather a surprising connection between these models and our model. To understand this, we describe each models in detail.

In the categorization model, categories (sets of alternatives) are compared by a shading relation, $\rhd$. For instance, the presence of salad dishes in the menu shades pasta dishes, or the presence of hamburgers shades other types of sandwiches. A DM considers only alternatives belonging to undominated categories according to the shading relation. That is, the alternative x will be eliminated in S if there exists $S_1, S_2 \subseteq S$ such that category S_1 shades category S_2 and x belongs to S_2. We can write the first stage as in our terminology:

$$\Gamma_{\rhd}(S) = \{x | \text{ there are no } S_1, S_2 \subseteq S \text{ such that } S_1 \rhd S_2 \ni x\}$$

Then the DM maximizes an asymmetric and complete binary relation among all surviving alternatives. Manzini and Mariotti (2012) shows that the categorization model, which is studied extensively in psychology, is characterized by a single axiom, so-called Weak-WARP.

In the rationalization model, a DM must rationalize every choice they make. A rationale can be intuitively understood as a story that states that some options are better than others. For example, someone might prefer watching a movie to visiting a relative in the hospital, but she cannot find a plausible story to justify watching the movie, hence visits the relative. In this model, the DM has a set of rationales/norms (R_i, represented as a binary relation) to rationalize her choices. For each choice set S, she identifies alternatives that are optimal according to at least one of her rationales. The first stage can be written as follows:

$$\Gamma_{\{R_i\}}(S) = \{x | \text{ there exits } R_i \text{ such that } xR_iy \text{ for all } y \in S \setminus x\}$$

Then, as in the categorization model, she picks an alternative which is preferred to all surviving alternatives according to an asymmetric binary relation. Surprisingly, this model is also characterized by Weak-WARP (Cherepanov et al. (2013)). This result is surprising since the first stages in these models look distinct. While rationalization and categorization models differ in their motivations and underlying stories (which makes them conceptually different and distinguishable), they are indistinguishable by choice data alone.

To connect these models to ours, first note that both $\Gamma_{\triangleright}$ and $\Gamma_{\{R_i\}}$ satisfy our competition filter property. Now we show that any competition filter can be written as in both $\Gamma_{\triangleright}$ and $\Gamma_{\{R_i\}}$. Take a competition filter Γ. We first define set of rationales: $x \; R_{(x,S)} \; y$ if $y \in S \setminus x$ and $x \in \Gamma(S)$. We then define a shading relation; $S \triangleright \{y\}$ if $y \notin \Gamma(S)$. It is routine to check the corresponding $\Gamma_{\triangleright}$ and $\Gamma_{\{R_{(x,S)}\}}$ are equal to Γ. While their stories are specific, since the degree of freedom provided by $\{R_{(x,S)}\}$ and $\triangleright$, three models generate the same behavior in the first stage. They thus are indistinguishable from each other on the basis of choice data alone even though they capture very different positive models of behavior.

The lack of consideration of some alternatives plays a relevant role in several papers. Masatlioglu and Ok (2014) propose a reference-dependent model where each status quo generates a psychological constraint set of alternatives that the DM is prepared to choose from given that status quo.[15] Masatlioglu and Nakajima (2013) study behavioral search by utilizing the idea of consideration sets. In this model, the consideration set dynamically evolves during the course of search. Caplin and Dean (2011) also study behavioral search by utilizing "choice process data," which include what the decision maker would choose at any given point in time if she were suddenly forced to quit searching. Eliaz et al. (2011) consider a model where the consideration sets of DMs are directly observed. Salant and Rubinstein (2008) propose a model where the DM considers only the top N elements according to some ranking and chooses her most preferred element from that restricted set.

4 Conclusion

Consumers generally do not consider all the available alternatives; they intentionally or unintentionally ignore some of the alternatives and focus on a limited number of alternatives. In this paper, we relax the implicit full consideration assumption of the standard choice theory to allow for the choice with limited consideration, where we

[15] A recent paper by Dean et al. (2014) investigate the implication of competition filter on reference-dependent behavior.

allow “feasible alternatives” and “considered alternatives” to differ for a given choice problem.

Marketing and finance literatures argue that the abundance of alternatives is the basic motive for limiting the consideration set. It is well documented that different types of filters have been used by the consumers to limit their consideration sets. Motivated by real-life examples observed different situations, we provide characterization for a general filter that includes many of these examples: if a consumer considers an alternative among a large set of option, she will still continue to considering the same alternative when some alternatives become unavailable. Moreover, we add another general intuitive property that is also consistent with observed behavior on different markets: if an alternative that the consumer does not consider becomes unavailable, his consideration set will not be affected.

Although the consideration sets are not observable, our axiomatic approach enables to distinguish and identify preferences and consideration by observing DM’s choices. We show our choice with limited consideration is capable of explaining behavioral anomalies that look puzzling under standard choice theory. Additionally, we believe these insights may help companies to develop new marketing strategies such that their products will attract attention by the consumers.

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A Appendix

A.1 Overwhelming Choice: Choice Correspondences

In the following section we show that the approach can be extended to choice correspondences. Here, we generalize the main characterization and revealed preference results from the paper, the characterization of our overwhelming choice, by allowing choices to be multi-valued and choice sets be arbitrary.

In this section we consider the general case where X is the (possibly infinite) choice set. $\mathcal{X}$ is the set of all non-empty finite subsets of X. Similarly, a choice correspondence will be given by $C : \mathcal{X} \to X$, such that $C(S) \subseteq S$ for every $S \in \mathcal{X}$. So far, c has been used to represent a choice function; here, we use C to represent a choice correspondence. When choices are multi-valued, instead a linear order we need to consider weak orders to allow for the possibility of an indifference relation. Now we let $\succsim$ be a weak order on X. We show it is possible to characterize the overwhelming choice model in this (more general) setting with only a few changes that account for the possibility of a "revealed indifference".

In section 2 we showed that the only behavioral postulate that characterizes the overwhelming choice model was WARP-CO. In this section, we characterize the OC model for choice correspondences by first adding a consistency axiom for multivalued choices (which will characterize the indifference relation with limited consideration); and second by modifying WARP-CO.

The new condition, which guarantees that the indifference relation is identifiable, requires multi-valued choices to also be consistent across choice sets. This new axiom, called Weak Revealed Indifference (WRI), does not allow for choice reversals involving only one element when two elements have been chosen from a larger menu. In other words, if WARP is violated from S to T[16], then the intersection between $C(T)$ and $C(S)$ must be empty.

Referring back to the intuition for WARP-CO, if there is more than one element, x^*, y^* satisfying the property for WARP-CO for a particular set, S, then those two elements must satisfy (or not) WARP-CO together for any set that contains both of them. In other words, once x^* and y^* satisfy the condition from WARP-CO for S, for any other set T such that $\{x^*, y^*\} \subseteq T \subset S$ if x^* satisfies the WARP-CO condition so does y^*.

> **Weak Revealed Indifferences (WRI).** If for $\{x, y\} \subseteq T \subset S$, $x, y \in C(S)$, then $x \in C(T)$ implies $y \in C(T)$.

One consequence of WRI is that we are be able to distinguish between (strict) revealed preference and revealed indifference from choice data, as shown by the following lemma.

[16]That is, $T \subset S$ and $y \in C(S) \cap T$, and $y \in C(T)$, then $C(S) \cap C(T) = \emptyset$.

Lemma 2. *Let C satisfy WRI. Then $x \in C(T)$ and $x \notin C(T')$ for some $S \subseteq T$ implies $C(T') \cap C(T) = \emptyset$.*

Proof. Suppose there exists $y \in C(T) \cap C(T')$, then by WRI we have $x \in C(T')$, since $x, y \in C(T)$ and $y \in C(T')$ which would be a contradiction. $\square$

The second axiom that we introduce to characterize the overwhelming choice model for correspondences is called No Cyclic Choice Reversals. Abusing terminology, we say that there is a choice reversal from x to y whenever there is a violation of WARP, i.e. $x \in C(T)$ and $y \in C(T')$ for $T' \subset T$. No Cyclic Choice Reversals is a stronger condition than WARP-CO, since it guarantees not only that WARP-CO is satisfied, but also, that once there is a choice reversal form x to y, we cannot find a chain of pairwise choice reversals that would indirectly imply the a choice reversal from x to y (i.e. from x to z_0, from z_0 to z_1,..., and from z_n to y).

It is straightforward to see that NCCR implies WARP-CO for choice functions.

> **No Cyclic Choice Reversal (NCCR).** Consider two families of sets $\{S_i\}$ and $\{T_i\}$ such that $T_i \subseteq S_i \subseteq X$ for $i = 1, \ldots, n$. If $C(T_{i+1}) \cap C(S_i) \neq \emptyset$ for all $i \leq n-1$ and $C(T_i) \cap C(S_i) = \emptyset$ for some i then $C(T_1) \cap C(S_n) = \emptyset$

Again, we define two binary relations: P and I. The former will capture the revealed preferences, the later captures the revealed indifference.

Definition. *Given a choice correspondence C define two binary relations, P and I as follows.*

1. *xPy if there exists $\{x, y\} \subseteq S \subset T$ such that $x \in C(S)$ and $y \in C(T) \setminus C(S)$.*

2. *xIy if there exists $\{x, y\} \subseteq S$ such that $\{x, y\} \subseteq C(S)$.*

First of all, note that if a choice correspondence satisfies our two axioms, we have that P and I are disjoint.

Proposition 2. *If C satisfies NCCR and WRI, then $P \cap I = \emptyset$.*

Proof. Let xPy, then there exists $\{x, y\} \subseteq S \subset T$ such that $x \in C(S)$ and $y \in C(T) \setminus C(S)$. Suppose there exists $T' \in \mathcal{X}$ such that $x, y \in C(T')$. By lemma 2, $C(T) \cap C(S) = \emptyset$, and $\{x, y\} \subseteq T'$, then $C(\{x, y\}) = \{x, y\}$ by WRI. And by NCCR, since $C(\{x, y\}) \cap C(S) \neq \emptyset$, we have $C(\{x, y\}) \cap C(T) = \emptyset$, which is a contradiction. So there does not exists such a T', and therefore $\neg(xIy)$.

Let xIy, then there exists $S \supseteq \{x, y\}$ such that $x, y \in C(S)$. By WRI we must also have $C(\{x, y\}) = \{x, y\}$. Suppose there exists $\{x, y\} \subseteq S \subset T$ such that $x \in C(S)$ and $y \in C(T) \setminus C(S)$. Then by lemma 2 we must have $C(S) \cap C(T) = \emptyset$. And $C(S) \cap C(\{x, y\}) = x$ implies $C(T) \cap C(\{x, y\}) = \emptyset$, which is a contradiction since $y \in C(T)$ and $C(\{x, y\}) = \{x, y\}$. Therefore no such $S \subset T$ exists, thus $\neg(xPy)$. $\square$

Lemma 3. *Let $R = P \cup I$, then xRy if and only if there exists $\{x, y\} \subseteq S \subseteq T$ such that $x \in C(S)$ and $y \in C(T)$.*

Proof. This follows from the definitions of P and I. $\square$

Hence we can see that the two axioms are equivalent to not being able to find two conflicting choice reversals. In parallel to the function case, this will imply that once we take the transitive closure of R, we will not have cycles.

Proposition 3. *C satisfies NCCR and WRI if and only if for any set of elements in X, $\{x_i\}_{i=1}^n$, such that $x_n R x_{n-1} R \dots R x_2 R x_1$ imply $\neg(x_1 P x_n)$.*

Proof. ($\Rightarrow$) Consider $x_i \in X$ for $i = 1, ..., n$, such that $x_n R x_{n-1} R \dots R x_2 R x_1$. Then by the definition of R, there must exists for each $i = 2, \dots, n$ sets and subsets $S_i \supseteq T_i$ with $x_i \in C(S_i)$ and $x_{i-1} \in C(T_i)$ (see lemma 3).

Suppose $x_1 P x_n$, then there exists $\{x, y\} \subseteq S' \subset T'$ such that $x_1 \in C(S')$ and $x_n \in C(T') \setminus C(S')$. By WRI, $C(S') \cap C(T') = \emptyset$. Let $S_1 = S'$ and $T_1 = T'$, then by NCCR $C(S_n) \cap C(T_1) = \emptyset$, but $x_n \in C(S_n)$ by definition, and $x_n \in C(T_1)$ by $x_1 P x_n$, a contradiction. Therefore $\neg(x_1 P x_n)$.

($\Leftarrow$) Let $x, y \in C(T)$ and $x \in C(S)$ for some $S \supseteq \{x, y\}$. Then we have yIx, which by definition of R implies yRx. If $y \notin C(S)$ then by definition of P we have xPy, but this is a contradiction since yRx implies $\neg(xPy)$ by the condition. So C satisfies WRI.

Let $S_i \subseteq T_i$ such that $x_i \in C(S_i)$ and $x_{i-1} \in C(T_i)$ for $i = 2, \dots, n$ and $x_1 \in C(S_1)$. So we have $C(S_i) \cap C(T_{i+1}) \neq \emptyset$ for all $i = 2, \dots, n$. This implies $x_n R x_{n-1} R \dots R x_2 R x_1$. Now we prove the contrapositive, let C fail NCCR. WLOG let $x_n \in C(T_1)$ and $C(T_1) \cap (S_1) = \emptyset$, so $C(S_n) \cap C(T_1) \neq \emptyset$, and for one of the S_i, T_i, $C(S_i) \cap C(T_i) = \emptyset$. Then we have $x_n R x_{n-1} R \dots R x_2 R x_1$, and since $x_n \in C(T_1)$ and $x_n \notin C(S_1)$, and $x_1 \in C(S_1)$, by definition of P $x_1 P x_n$. This fails the condition that $x_n R x_{n-1} R \dots R x_2 R x_1$ implies $\neg(x_1 P x_n)$. $\square$

Proposition 3 implies that we can take the transitive closure of R, R_T, without creating any conflict. We then show that any completion of R_T will represent C by choosing an appropriate Γ satisfying the Competition Filter property. The following theorem shows that OC behavior when allowing for choice correspondences is completely characterized by the two axioms NCCR and WRI.

Theorem 2. *A choice correspondence C is an overwhelming choice if and only if C satisfies NCCR and WRI.*

Proof. ($\Rightarrow$) First we show necessity of the two axioms. Let C be an overwhelming choice represented by $(\succsim, \Gamma)$, where Γ is a competition filter.

To prove NCCR, let $T_i \subseteq S_i$ be a set of menus such that $C(S_i) \cap C(T_{i+1}) \neq \emptyset$. Without loss let $C(S_1) \cap C(T_1) = \emptyset$. Let $x_i \in C(S_i)$ and $y_i \in C(T_i)$ be elements of the respective choice sets.

Since C is an overwhelming choice, the information $C(S_1) \cap C(T_1) = \emptyset$, $C(S_i) \cap C(T_{i+1}) \neq \emptyset$, and $C(S_i) \cap C(T_{i+1}) \neq \emptyset$ tells us $x_i, y_i \in \Gamma(T_i)$ for all i and therefore we can conclude

$$\begin{aligned} y_1 &\succ x_1, \\ y_i &\succsim x_i \ \forall i \\ x_{i+1} &\sim y_i \ \forall i \end{aligned}$$

Therefore we have $y_n \succsim x_n \sim y_{n-1} \succsim \cdots \succsim y_2 \succsim x_2 \sim y_1 \succ x_1$. Since $\succsim$ is a weak order, we must have $y_n \succ x_1$. For any $z \in C(S_n)$, $z \sim y_n$ since C is a OC. So $z \succ x_1$ and $z \succ w$ for any $w \in C(T_1)$. This implies that for all $w \in C(T_1)$, $w \notin C(S_n)$. Similarly for any $z \in C(T_1)$, $z \sim x_1$ and since C is a OC represented by $(\succsim, \Gamma)$, $w \succ z$ for all $w \in C(S_n)$, and we get $z \notin C(T_1)$ since c is a OC. Therefore $C(S_n) \cap C(T_1) = \emptyset$.

Now we prove the necessity of WRI. Let c be an overwhelming choice represented by $(\succsim, \Gamma)$. Suppose $x, y \in C(S)$ for some $\{x, y\} \subseteq T \subset S$. Then $x, y \in \Gamma(S)$ and since Γ is a competition filter, $x, y \in \Gamma(T)$. Given that $x, y \in \Gamma(S) \cap \mathcal{C}(S)$, and there is a weak order $\succsim$ such that $C(S) = \max_{\succsim} \Gamma(S)$ for all S, we must have $x \sim y$. Let $x \in C(T)$, then for all $z \in \Gamma(T)$, $x \succsim z$. By transitivity $y \succsim z$ for all $z \in \Gamma(T)$, since $x \sim y$ and $y \in \Gamma(T)$. Therefore $y \in C(T)$. Which means that c satisfies **WRI**.

($\Leftarrow$) By Proposition 3, the transitive closure of R_T is well-defined. Let $\succsim$ be any completion of R_T. Define

$$\Gamma^m(S) = \{x \in S |\ x \in C(T) \text{ for some } T \supseteq S\}$$

We now show that $(\Gamma^m, \succsim)$ represents C.

First let $x \in C(S)$, we want to show that x is $\succsim$-maximal in $\Gamma^m(S)$. x belongs to $\Gamma^m(S)$ by construction. Let $y \in \Gamma^m(S)$, then $y \in C(T)$ for some $T \supseteq S$, therefore xRy by definition of R. Since $R_T \subseteq \succsim$, $x \succsim y$ follows by construction. Therefore, x is $\succsim$-maximal in $\Gamma^m(S)$.

Now let $x \notin C(S)$. To obtain a contradiction, assume $x \in \Gamma^m(S)$. This implies $x \in C(T)$ for some $T \supseteq S$. Let z be in $C(S)$. Hence, we have zPx, so $z \succ x$. This means that x is not $\succsim$-maximal in $\Gamma^m(S)$. Therefore, we can conclude that $(\Gamma^m, \succsim)$ represents c. $\square$

Similarly to the definition of revealed preference for choice functions, we now define revealed (strict) preference and revealed indifference for choice correspondences.

Definition. *Let C be an overwhelming choice correspondence and that there are k different attention filter, weak orders representing C;*

$$(\Gamma_1, \succsim_1), (\Gamma_2, \succsim_2), \ldots, (\Gamma_k, \succsim_k)$$

1. *x is **revealed preferred** to y if $x \succsim_i y$ for all i.*

2. *x is (strictly) **revealed preferred** to y if $x \succ_i y$ for all i.*

*3. x is **revealed indifferent** to y if $x \sim_i y$ for all i.*

The next proposition states the revealed preference result for choice correspondences.

Proposition 4. *Let $(\Gamma, \succsim)$ represent c. Let R_T be the transitive closure of R. Let C be an overwhelming choice correspondence. Then x is revealed preferred to y ($x \succsim_R y$), if and only if xR_Ty.*

Proof. ($\Rightarrow$) See the proof of Theorem 2.

($\Leftarrow$) Let xR_Ty, then $z_1, \ldots, z_n$ such that $x = z_1Rz_2R\ldots Rz_n = y$ (possibly $z_2 = y$, in which case xRy). For any z_iRz_{i+1}, there exists $\{z_i, z_{i+1}\} \subseteq S_i \subseteq T_i$ such that $z_i \in C(S_i)$ and $z_{i+1} \in C(T_i)$. Since Γ_i is an attention filter, $z_i, z_{i+1} \in \Gamma(S_i)$; and $z_i \in C(S_i)$ implies that z_i is $\succsim$ −maximal in $\Gamma(S')$, i.e. $z_i \succsim z_{i+1}$ because C is a OC correspondence. Therefore $x = z_1 \succsim z_2 \cdots \succsim z_n = y$, and by transitivity of $\succsim$, $x \succsim y$ follows. Therefore $R_T \subseteq \succsim$. □

Corollary 2. *Let I_T and P_T be the symmetric and asymmetric components of R_T respectively.*

(i) x is revealed indifferent to y if and only if xI_Ty

(ii) x is (strictly) revealed preferred to y if and only if xP_Ty.